

Is this the end of dark energy?

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It is thought that the current phase of accelerated expansion of the Universe is driven by a hypothetical fluid with negative pressure, dubbed dark energy. This fluid, of unknown nature, would correspond to approximately 70% of the energy content of the Universe. The cosmological constant, which may be associated with the zero point energy of the quantum fields, is the strongest candidate for dark energy, despite the problems that it brings to the cosmology-particle physics interface. The main alternative to the vacuum energy is a dynamical scalar field. From the observational viewpoint it is extremely hard, if not impossible, to decide which of these scenarios is correct, if one of them is. The data shows only that the Universe is expanding at an accelerated rate but does not reveal which causes this acceleration, dark energy or something else. However, although unknown, dark energy is not immune to the laws of physics. Such an exotic fluid must satisfy the bounds imposed by the laws of thermodynamics which have a strong experimental basis. Here we investigate the limits imposed by thermodynamics to a dark energy fluid. We obtain the heat capacities and the compressibilities for a dark energy fluid. These thermodynamical variables are easily accessible experimentally for any terrestrial fluid. The thermal and mechanical stabilities require these quantities to be positive. We show that such requirements forbid the existence of a cosmic fluid with negative constant EoS parameter which excludes vacuum energy as a candidate to explain the cosmic acceleration. We also show that the current observational data from SN Ia, BAO and $H(z)$ are in conflict with the physical constraints that a general dark energy fluid with a time-dependent EoS parameter must obey which can be interpreted as an evidence against the dark energy hypothesis. Although our result excludes the vacuum energy, a geometrical cosmological term as originally introduced by Einstein in the field equations remains untouched.

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Since the standard cold dark matter model (SCDM) was discarded by the Type Ia Supernovae observations, which point to a present day accelerated Universe[1, 2], physicists have been challenged to find a model that agrees with the data and, at the same time, rely on a solid theoretical basis. This task is not easy. Usually, physicists start with the simplest model. Thus, the first attempt was to reintroduce the cosmological constant Λ to Einstein's field equations. A positive Λ term acts in the motion equations as a constant repulsive force and, therefore, can speed up the Universe at large scales. The so-called Λ CDM model is able to explain most of the current observational data and has a strong theoretical appeal since it may be linked with the zero point energy of all quantum fields filling the Universe. Physicists would have given their verdict in favor of the cosmological constant if it did not suffer from a serious problem: the value of the vacuum energy density obtained by observations differs from the value provided by quantum field theory by at least 60 orders of magnitude [3–5]. It

is very difficult to handle this huge discrepancy. If we think in terms of a net cosmological constant as the sum of a bare geometrical Λ term with the quantum vacuum energy density to explain such small value, this will generate a fine-tuning problem: the absolute value of the geometrical and matter contributions to the net cosmological constant must be extremely close. Also, symmetry arguments are not enough to explain the small value of the vacuum energy observed today. The lack of a reasonable explanation for the cosmological constant problem has led physicists to explore other routes to explain the observations. Scalar fields (quintessence) stand out among the alternatives to the cosmological constant since they provide a link between particle physics and cosmology. The energy density of such a form of scalar matter must evolve with time but should mimic a cosmological constant to be compatible with the data. However, the majority of scalar fields that adjust the data have no foundations on particle physics and are somewhat artificial. In fact, vacuum energy and quintessence are not the only possibilities to explain the cosmic acceleration. Other fluids with negative pressure called generally dark energy (DE) [6] and modifications of gravity theory [7] are also in the game.

By assuming that general relativity is the right theory of gravitation, the DE pressure must be sufficiently

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negative to make the sum $\rho + 3p$ negative in order to produce an accelerated expansion of the Universe. Dark energy is frequently characterized by the equation of state (EoS) parameter (the ratio between its pressure and its energy density) $w = p/\rho c^2$ which can be constant or time-dependent. Such a phenomenological approach encompasses vacuum energy ($w = -1$), scalar fields ($-1 \leq w \leq 1$) [8], phantom fields ($w < -1$) [9] and many other forms of exotic matter. If, in fact, the acceleration of the Universe is due to some type of dark energy, the big issue that must be answered is: what is dark energy? Vacuum energy, some type of scalar matter or even some form of more exotic matter? Observational data are not enough to allow us decide between the different types of dark energy since most of proposed models are able to adjust the data seamlessly. We therefore need to go deeper in theory in order to achieve a better understanding of the mechanisms behind the cosmic acceleration. In this direction, thermal physics is of particular importance in dark energy studies. The laws of thermodynamics are based on experimental evidence and apply to all types of macroscopic system. Unlike classical mechanics or electromagnetism, thermodynamics does not predict specific numerical values of observables. Thermodynamics sets limits on physical processes. The power of thermodynamics is its generality. Therefore, exploring the thermodynamic behavior of the cosmic fluids that pervade the Universe may be a line of attack for unveiling the nature of the contents of the Universe, particularly the hypothetical DE fluid. For example, the positiveness of the entropy may be one of the main weapons for imposing bounds on the EoS parameter of dark energy [10]. Applying the laws of thermodynamics to the dark energy fluid can help us to constrain or even to rule out dark energy models. Below we perform this task.

I. THERMODYNAMICS OF THE COSMIC FLUIDS

Consider an expanding, homogeneous and isotropic Universe filled by matter (baryonic and dark), described by a pressure-less perfect fluid ($w = 0$), radiation, described by a perfect fluid with a EoS parameter $w = 1/3$ and dark energy, described by a perfect fluid with a EoS parameter $w = p/\rho c^2$. Homogeneity and isotropy imply that all physical distances scale with the same factor $a(t)$, called the scale factor of the Universe. Thus, the physical volume of the Universe at a given time is $V = a^3(t)V_0$ ¹. In such a model the internal energy of the i -th fluid component can be written as

$$U_i = \rho_i c^2 V. \quad (1)$$

Assuming a reversible adiabatic expansion, the first law of thermodynamics

$$T_i dS_i = dU_i + p_i dV, \quad (2)$$

leads to so-called fluid equation,

$$d \ln \rho_i + 3(1 + w_i) d \ln a = 0, \quad (3)$$

which expresses the energy-momentum conservation. Assuming that the density is a function of the temperature and volume, i. e., $\rho_i = \rho_i(T_i, V)$, the fact that dS_i is an exact differential implies that [11]

$$d \ln T_i = -3w_i d \ln a, \quad (4)$$

or, using (3) to eliminate w_i ,

$$d \ln T_i = d \ln \rho_i + 3 d \ln a. \quad (5)$$

Integrating the temperature law (5) we get

$$\frac{T_i}{T_{i,0}} = \frac{\rho_i}{\rho_{i,0}} a^3. \quad (6)$$

or, in a more suggestive form

$$\frac{1}{w_i} \frac{p_i V}{T_i} = \frac{1}{w_{i,0}} \frac{p_{i,0} V_0}{T_{i,0}} = \text{constant}. \quad (7)$$

The above equation generalizes the ideal gas law for a time dependent EoS parameter. Finally, the fluid energy can be written in terms of the temperature as

$$U_i = U_{i,0} \frac{T_i}{T_{i,0}}. \quad (8)$$

In what follows, we derive the expressions for the heat capacity, compressibility and the thermal expansibility. These thermodynamical derivatives are easily accessible experimentally for any terrestrial fluid. The heat capacity and the compressibility of the fluid are related, respectively, with thermal stability and mechanical stability and must be greater than zero. Thus, we can use these variables to impose bounds on EoS parameter of DE.

A. The Universe's heat capacity

The classical thermodynamical definition of a fluid's heat capacity C_i is [12],

$$dQ_i = C_i dT_i, \quad (9)$$

where dT_i is the fluid temperature increase due to an absorbed heat $dQ_i = T_i dS_i$. The heat capacity of a fluid will differ depending on whether the fluid is heated at constant volume or at constant pressure. From the first law of thermodynamics (2) at constant volume (9) becomes

$$dU_i = C_{iV} dT_i, \quad (10)$$

¹ Here the index 0 will denote the present time value of an observable and we will adopt the convention $a_0 = 1$

where

$$C_{iV} = \left(\frac{\partial U_i}{\partial T_i} \right)_V, \quad (11)$$

is the fluid's heat capacity at constant volume. The heat capacity at constant pressure can be built up from the enthalpy,

$$h_i = U_i + p_i V, \quad (12)$$

in terms of which the first law of thermodynamics is written as

$$dQ_i = dh_i - V dp_i. \quad (13)$$

Thus, at constant pressure, (9) becomes

$$dh_i = C_{p_i} dT_i, \quad (14)$$

where

$$C_{p_i} = \left(\frac{\partial h_i}{\partial T_i} \right)_{p_i}, \quad (15)$$

is the fluid's heat capacity at constant pressure.

From equation (8), it is easy to show that

$$C_{iV} = \frac{U_{i,0}}{T_{i,0}} = \text{constant}, \quad (16)$$

for any component of the Universe. Since $p_i V = w_i U_i$, the enthalpy (12) becomes

$$h_i = (1 + w_i) U_i \quad (17)$$

and, from equations (8) and (4), we get

$$C_{p_i} = \left(1 + w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right) C_{iV}. \quad (18)$$

Since $U_{i,0} = \rho_{i,0} c^2 V_0$, the specific heat (the heat capacity per mass unit) at constant volume is

$$c_{iV} \equiv \frac{C_{iV}}{\rho_{i,0} V_0} = \frac{c^2}{T_{i,0}}. \quad (19)$$

For relativistic matter $T_{r,0} = 2.725 \text{ K}$ so that c_{rV} and c_{p_r} are of order of $\sim 10^{13} \text{ cal} \cdot \text{g}^{-1} \cdot \text{K}^{-1}$. Since the temperature of the others components must be smaller than the temperature of relativistic matter, the specific heat of the relativistic matter is a bottom limit for the Universe's specific heat. As expected, this result reveals that the Universe is a huge thermal reservoir. Unfortunately, despite easy experimental access for terrestrial fluids, we cannot isolate a cosmologically significant portion of the Universe, provide an enormous amount of heat and measure the temperature change of our Universe sample to get its specific heat experimentally.

B. Compressibility and expansibility

If we regard the volume as function of temperatures and pressures we have ²

$$dV = \sum_i \left[\left(\frac{\partial V}{\partial T_i} \right)_{p_i} dT_i + \left(\frac{\partial V}{\partial p_i} \right)_{T_i} dp_i \right]. \quad (20)$$

We define the thermal expansivity, which measures the volume thermal expansion at constant pressure, by

$$\alpha_i = \frac{1}{V} \left(\frac{\partial V}{\partial T_i} \right)_{p_i}, \quad (21)$$

and the isothermal compressibility, which measures the relative change of volume with increasing pressure at fixed temperature, by

$$\kappa_{T_i} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_i} \right)_{T_i}. \quad (22)$$

Analogously to the isothermal compressibility, we can define the adiabatic compressibility κ_{S_i} , if, instead of temperature, the entropy is kept fixed. It can be shown that the isothermal compressibility and the isothermal expansibility are related by

$$\frac{\alpha_i}{\kappa_{T_i}} = \left(\frac{\partial p_i}{\partial T_i} \right)_V, \quad (23)$$

and that the ratio between the adiabatic and the isothermal compressibilities are equal to the ratio between the heat capacities at constant volume and at constant pressure, i.e.,

$$\frac{\kappa_{S_i}}{\kappa_{T_i}} = \frac{C_{iV}}{C_{p_i}}. \quad (24)$$

Noting that $p_i V = w_i C_{iV} T_i$ and using (4) we obtain

$$\alpha_i = \frac{C_{iV}}{p_i V} \left(w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right). \quad (25)$$

From (23) is easy to show that

$$\kappa_{T_i} = \frac{\alpha_i V}{w_i C_{iV}}, \quad (26)$$

and from the above equation and (24) we have

$$\kappa_{S_i} = \frac{\alpha_i V}{w_i C_{p_i}}. \quad (27)$$

² Remember that we are assuming that the fluids evolve separately, that is, they do not exchange heat, as shown by eq. (3)

II. CONSTRAINTS ON DARK FLUID

Thermal stability requires that $C_{iV}, C_{pi} \geq 0$ and mechanical stability requires that $\kappa_{Si}, \kappa_{Ti} \geq 0$. Additionally, $C_{iV}, C_{pi}, \kappa_{Si}$, and κ_{Ti} are related by $C_{pi} = C_{iV} + \frac{TV\alpha_i^2}{\kappa_{Ti}}$ and $\kappa_{Ti} = \kappa_{Si} + \frac{TV\alpha_i^2}{C_{pi}}$ so that $C_{pi} \geq C_{iV}$ and $\kappa_{Ti} \geq \kappa_{Si}$. From (18) and (25) is easy to see that the above conditions are satisfied only if the fluid obeys the constraint

$$w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \geq 0. \quad (28)$$

Reference	w	$w - \frac{1}{3} \frac{d \ln w }{d \ln a} \geq 0 \quad \forall a \in [0, \infty)$
[13]	$\frac{w_0}{(1-b \ln a)^2}$	No
[14]	$w_f + \frac{\Delta w a_t^{1/\tau}}{a_t^{1/\tau} + a^{1/\tau}}$	No
[15]	$w_f w_i \frac{a^l + a_t^l}{w_i a^l + w_f a_t^l}$	No
[16]	$w_0 + w'_0(a - a^2)$	No
[17]	$w_0 + w'_0 \frac{a-1}{1-2a+2a^2}$	No
[18]	$w_0 + w'_0 \frac{a^\beta - 1}{\beta}$	No

TABLE I. Thermodynamic viability of some DE parametric models found in literature. Here, $w'_0 = (dw/da)_{a=1}$.

It is obvious that if w_i is constant, thermal and mechanical stability implies that $w_i \geq 0$ which rule out all negative pressure fluids with a constant EoS parameter including vacuum energy. Thus, if the accelerated expansion of the Universe is caused by a dark energy fluid, its EoS parameter must be time-dependent. In Table I we list some time-dependent guesses of the DE EoS parameter. Although in excellent agreement with the data, these phenomenological models do not satisfy the thermodynamical bound (28).

Now, let w and ρ_{DE} denotes, respectively, the EoS parameter and the density of the dark energy fluid. Since w must evolves with the time, we can use (3) to rewrite the inequality (28) as

$$3 + \frac{d \ln \rho_{DE}}{d \ln a} \leq -\frac{d \ln |w|}{d \ln a}. \quad (29)$$

Integrating both sides of the inequality above we get

$$\left| \frac{w}{w_0} \right| \leq \frac{\rho_{DE,0}}{\rho_{DE} a^3} = \frac{T_{DE,0}}{T_{DE}}, \quad (30)$$

where we have used (6). According to (4) the temperature of the fluid will increase (decrease) with the expansion of the Universe if $w < 0$ ($w > 0$). The constraint (30) reveals that an eternal accelerated expansion cannot be sustained. If $w < 0 \quad \forall a \geq 1$, $T_{DE}(a \rightarrow \infty) \rightarrow \infty$ and $|w(a \rightarrow \infty)| \rightarrow 0$ which means that the accelerated expansion will stop in the distant future. On the other hand, if the sign of w change in the course of the expansion, the Universe can enter and leave in a accelerated expansion phase depending on how many times the sign of w changes but cannot keep an acceleration expansion phase forever. This transient behavior imposed by thermodynamics is particularly important for formulation of the String/M theory since an eternal accelerated expansion implies that a conventional S-matrix cannot be built [19–21].

Now, at present time the inequality (28) becomes

$$w'_0 \geq 3w_0^2. \quad (31)$$

In order to check the compatibility of the observational data with the above thermodynamic constraint we follow the approach developed in [22] which is one of the less model dependent methods to probe the DE EoS time-dependence. This approach consists in assuming that the DE density admits a Taylor expansion in the range $(\tilde{a} - \epsilon_-, \tilde{a} + \epsilon_+)$, that is,

$$\begin{aligned} \rho_{DE}(a) = \rho_{DE}(\tilde{a}) + \frac{d\rho_{DE}}{da} \Big|_{a=\tilde{a}} (a - \tilde{a}) + \\ + \frac{1}{2} \frac{d^2 \rho_{DE}}{da^2} \Big|_{a=\tilde{a}} (a - \tilde{a})^2 + \dots \end{aligned} \quad (32)$$

and then use the conservation equation (3) as a recurrence formula to write the derivatives of ρ_{DE} in terms of the derivatives of w , i. e.,

$$\begin{aligned} \frac{d\rho_{DE}}{da} &= -\frac{3}{a}(1+w)\rho_{DE}, \\ \frac{d^2 \rho_{DE}}{da^2} &= \left[\frac{3}{a^2}(1+w) + \frac{9}{a^2}(1+w)^2 - \frac{3}{a} \frac{dw}{da} \right] \rho_{DE}, \\ &\vdots \end{aligned}$$

This approach allows us to constraint w and its derivatives at different redshifts simply changing the series expansion center \tilde{a} . Since for sufficiently small values of ϵ_{\pm} the second order approximation must work reasonably well, we will restrict our analysis up to the second order expansion of ρ_{DE} around $\tilde{a} = a_0 = 1^3$. Thus, choosing the expansion center at a_0 , the second order approximation of the DE density becomes,

$$\begin{aligned} \rho_{DE}(a) = \rho_{DE,0} \left\{ 1 + 3(1+w_0)(1-a) + \frac{1}{2} [3(1+w_0) \right. \\ \left. - 3w'_0 + 9(1+w_0)^2] (1-a)^2 \right\}. \end{aligned} \quad (33)$$

³ For example, taking the expansion center $\tilde{a} = a_0 = 1$ and $\epsilon_+ = 1/3$ is possible to cover the redshift range $0 \leq z \leq 2$.

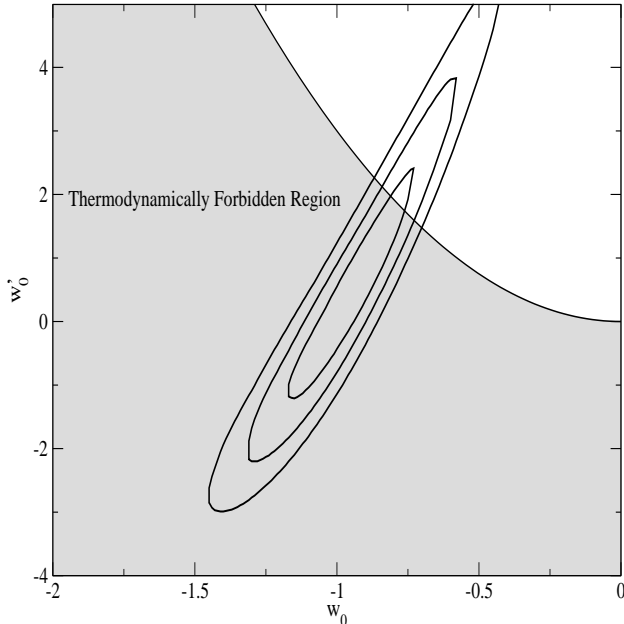


FIG. 1. The $w_0 - w'_0$ parametric space. The thermodynamically forbidden region corresponds to points in the space of phase for which the inequality (31) is not satisfied. The contours are drawn for $\Delta\chi^2 = 2.30, 6.17$ and 11.8 .

Figure 1 shows the observational constraints in $1, 2$ and 3σ on w_0 and w'_0 for a specially flat, homogeneous and isotropic Universe filled by relativistic matter, non-relativistic matter and a dark energy fluid described by (33) obtained from 580 supernovae data of Union 2.1 compilation, the six estimates of the BAO points given in Table 3 of Ref. [26] and the 28 measurements of the Hubble function $H(z)$ compiled by Liao *et al.* [24] and Farooq & Ratra [25] (see also reference therein). The present value of the Hubble parameter H_0 and the matter density parameter $\Omega_{m,0}$ were marginalized. For this data combination the best fit values are $w_0 = -0.96^{+0.22}_{-0.21}$ and $w'_0 = -0.33^{+2.00}_{-1.53}$ with the upper and lower values denoting the one parameter 1σ errors. As we can see, a large portion of the $w_0 - w'_0$ confidence regions lies in the unphysical region. This lack of sensitivity of the data to the physical constraint $w'_0 \geq 3w_0^2$ can be interpreted as an evidence against the dark energy hypothesis since if dark energy is the true cause of the accelerated expansion the data would not be in conflict with its physical properties. However, if dark energy does not exist the data would not be forced to feel its physical properties and would force the dark energy parameters to converge for the values that better approximates of the true piece behind the cosmic acceleration regardless of the physical bounds that a hypothetical dark energy fluid must obey.

III. FINAL REMARKS

What is causing the accelerated expansion of the Universe? Is it dark energy or is it that Einstein's general relativity that does not work at large scales? Physicists have worked on both fronts to answer that question. This is the old trial and error method. Particularly, on the dark energy side a large number of models have been proposed. The diversity of scenarios existing in the literature is due to the diversity of aspects to be studied beyond the EoS parameter time-dependence, such as fast variation, DE implications in the early Universe, and so on. Obviously, by adopting this line of attack, nobody expects to find the true causes of the accelerated expansion of the Universe. Instead it is expected to find clues for the right answer between the models' debris. In this article we believe we have taken a big step in understanding the current phase of accelerated expansion of the Universe. We have studied the thermodynamical aspects of an expanding, homogeneous and isotropic Universe filled by matter (baryonic plus dark), relativistic matter (radiation plus neutrinos) and a hypothetical dark energy. We have estimated the Universe's specific heat and examined the constraints imposed by classical thermodynamics on the dark energy. We have shown that the thermal and mechanical stability conditions forbid the existence of negative pressure fluids with a constant EoS parameter which excludes the vacuum energy as a candidate to explain the cosmic acceleration. We also show that the observational data are in conflict with the thermodynamic constraints that a general dark energy fluid with a time-dependent EoS parameter must satisfy. This result suggest that adding dark energy to the content of the Universe may not be the answer to the cosmic acceleration problem.

We must noting that, although our analysis excludes the vacuum energy, this does not represent the end of the cosmological constant. A bare geometrical Λ -term remains in the game if interpreted as a constant of the nature whose value must be determined by observations. However, what happens to the vacuum energy? Is it null? If so, why? We know that the vacuum energy has a significant role in the quantum world but, should it play any significant role in the Universe at large scales? Can we add the quantum vacuum energy so naively to classical general relativity field equations? We know that the fluid description works for relativistic and non relativistic matter but, can we describe the vacuum energy simply as a fluid? Perhaps only a quantum theory of gravity can provide an answer to these issues. Beyond the issues raised above a question still remains: is the geometrical cosmological constant the explanation to the accelerated expansion? The Λ -term certainly is the simplest solution but nobody can guarantee that it is the true answer. Thus, finding out deviations of the cosmological term will remain as one of the hottest theoretical investigation lines concerning cosmic acceleration. If the dark energy is out of the game, approaches such as the kinematic method developed in [27] can be a useful tool

to search for such deviations.

Obviously, we can sacrifice the thermodynamical stability conditions to keep the dark energy hypothesis alive. For example, if we relax the mechanical stability condition, but keep the thermal stability we have, from (28), that $w \geq -1$ saving vacuum energy and quintessence. If additionally we give up the thermal stability, phantom fields ($w < -1$) are also allowed⁴. However, we do not think that this is a good way to address the problem.

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⁴ Thermodynamical systems that do not meet the stability conditions are inhomogeneous. Globular clusters [28] and black holes

[29] are examples of such systems.